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Investigating shape and space in mathematics: a case study

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In this investigation I focused on a process of activities of a particular group of teachers and learners. Basic skills in mastering mathematical concepts were addressed. The focus was on concept formation in the geometry classroom. The methodology involved an educational case study as a form of inquiry to investigate spatial sense. Evidence was obtained regarding mathematics teachers' and mathematics learners' knowledge of space and shape. Problems experienced in concept formation in geometry were investigated and analysed. An account is provided of how teachers and learners responded to problems related to space and shape. Information about the mathematical performance of a group of mathematics teachers and mathematics learners is organised, interpreted, and evaluated.

Introduction

Mathematics mastery is a growing concern in South Africa. It is scientifically proven, well documented, and generally accepted that South Africa does not rate highly internationally in mathematics performance (EFA, 2004, SACMEQ II, 2004). Research has revealed that South African learners' achievement in numeracy is among the worst in the world (Govender, 2004:12). A need to better represent what mathematics is about and to popularise the discipline has been growing over the last few decades (NCTM,1989a; NCTM, 1989b; NCTM, 1990; SACMEQ I,1998; SACMEQ II, 2004, EFA, 2004, Van de Walle, 2004; Boaler, 2001; Adler, 1999; Govender, 2004:12, George, 2005:2, Theunissen, 2005:64, HSRC, 2000:1). In this study problems relating to concept formation and spatial visualisation in geometry were investigated. The problems were based on the mathematics content of the National Curriculum Statement (NCS) (NCS, 2003). Specific geometry content was identified involving skills in reasoning. Aspects of quadrilaterals, symmetry, proportions, and three-dimensional figures were evaluated by means of a simple measuring instrument. Mastering aspects of space and shape were addressed against a background of broad mathematical concepts.

Research question and aim of the study

Aspects of concept formation and spatial visualisation in geometry were investigated. The research problem was formulated as follows:

> What are some of the problems experienced by teachers and learners in space and shape in the geometry classroom?

In order to seek answers to the problem, the performance of mathematics teachers and learners was investigated and compared. Answers were sought to the following questions:

- What are teachers' and learners' conceptual understanding of shape and space?

- What are teachers' and learners' achievement in space and shape?
- What are the gender and achievement relationships of teachers and learners?

The general aim of the study was therefore to provide insight into problems experienced in space and shape in the mathematics classroom. More specifically insight was sought into the relationship between teachers' and learners' mathematics achievement in space and shape.

Shape and space in the mathematics learning area

Mathematics, according to Van de Walle (2004:4), focuses on categories such as patterns and relations, has a language of its own which requires the use of precise mathematical terms and symbols and is an organised field of knowledge with interrelated and interdependent content and process standards or strands. The National Council of Teachers of Mathematics (NCTM, 2000: 28-67) identifies various principles and standards, including five content standards (number and operation, algebra, geometry, measurement, and data analysis and probability) and five process standards (problem solving, reasoning and proof, connections, communication, and representation). The Revised National Curriculum Statement (RNCS) (RNCS, 2002:4) uses similar broad categories, namely patterns and relationships; symbols and notations of a specialised language; fields of knowledge or content standards (learning outcomes) and specialised skills or process standards (assessment standards). Essential knowledge components [learning outcomes, (RNCS, 2002)] include numbers, operations and relationships; patterns, functions and algebra; space and shape (geometry); and data handling. According to the National Curriculum Statement (NCS) learning outcomes are intended results of learning and teaching, describing knowledge, skills and values that learners should acquire (NCS, 2003:7). The way in which these outcomes are achieved is by means of process standards (NCTM, 2000) or assessment standards (RNCS, 2002).

The RNCS (2002:5) identifies the following reasoning patterns (skills) for the learning and teaching of mathematics: representation and interpretation; estimation and calculation; reasoning and communication; problem posing; problem solving and investigation; describing and analysing. The list is further expanded when unique features are outlined and corresponding skills are identified, such as visualising, ordering, estimating, interpreting, comparing, classifying, analysing, synthesising, etc. (RNCS, 2002:6). It is interesting to note that the list of reasoning patterns or skills includes a total of 31 skills associated with numbers, data, space and shape; problem solving, and investigating patterns and relationships.

These skills are subsequently reformulated within the identified learning outcomes as assessment standards, where terms are listed, such as interpret, choose effectively, identify, convert, investigate, conjecture, justify, generalise, apply, and others (NCS, 2003:16-83). Van de Walle (2004:13) refers to such terms as 'science terms' when he uses similar terms such as explore, solve, represent, formulate, discover, construct, verify, explain, predict, develop, describe, and use. Assessment standards describe knowledge and skills and guide conceptual progression (NCS, 2003:7). Assessment standards indicate what a learner should know and be able to demonstrate at a specific grade, thus embodying the knowledge, skills and values required to achieve the learning outcomes.

Geometry (space and shape) is an important knowledge component of both official documents of the National Department of Education in South Africa (RNCS, 2002; NCS, 2003). One of the learning outcomes of both documents comprises working with space and shape (geometry). This particular learning outcome indicates that a mastery of space and shape will be demonstrated by learners if they are able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions (RNCS, 2002), as well as able to analyse and explain properties of two-dimensional and three-dimensional shapes with justification (NCS, 2003). To achieve this, corresponding assessment standards on Grade 10 level reflect an understanding of volume and surface area of right prisms and cylinders and reasoning abilities to produce conjectures related to triangles, quadrilaterals and other polygons (NCS, 2003:32). Learners are further required to investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangle, the kite, parallelogram, rectangle, rhombus, trapezoid and square).

If views of how learning takes place are examined, no one view of learning will be completely effective for all learners. Cognitive and constructivist learning theories, respectively, emphasise the role of metacognition or the self-monitoring of learning and thinking (Shepard, 2000:4) and the idea that knowledge is constructed through a process of creating personal meaning from new information and prior knowledge within realistic settings (McMillan, 2004:12). Educators assist learners to link new knowledge to existing knowledge and develop instructional techniques that would facilitate cognitive growth and change. Key cognitive processes are examined in assessing a particular concept and hence instructional methods are designed to help learners develop these processes. Van de Walle (2004:36) views teaching in this regard as assisting learners to construct knowledge through problem posing and engaging learners in mathematical discourse so that they may examine their new assumptions about mathematics. Research identifies trends such as cognitive development towards the mastering of advanced mathematical concepts and processes (Hammill & Bartell, 1995:255; Clements & Battista, 2001), and a shift from a procedural approach (calculation accuracy) to a conceptual approach (the sensible application of procedures) (Brown, 1999:3). Contemporary views underline the idea that the cognitive prerequisites for mastering mathematics involve more than traditional computation skills. Cognitive development, according to Troutman and Lichtenberg (2003:10), involves internal representations (internal development) and external representations. They argue that conceptual learning occurs if children build internal representations composed of networks of concepts and relationships that mirror desired external representations. They warn against explaining, showing and telling, implying that this is procedural teaching (or rote learning which they regard as primarily a didactic mode). However, they agree that there are historical methods and techniques that need to be preserved for future generations (Troutman & Lichtenberg, 2003:2).

Mastering space and shape concepts in geometry offers opportunities to practice logical reasoning and to acquire abilities in various reasoning patterns. Troutman and Lichtenberg (2003:407) argue that through the listing of properties and classifications learners begin to build concepts enabling them to develop the spatial sense to function in their environment. Reasoning skills are necessary to advance from a procedural to a conceptual approach. In geometry various geometric approaches may be utilised towards such advancement, including topological geometry (or the effect of changes on certain attributes or curves); projective geometry (or viewing objects from different perspectives); Euclidean geometry (or bodies of knowledge consisting of statements justified by proofs, which depend on mathematical axioms and an underlying logic) (Ernest, 1991:6-7); informal geometry (or explorative, hands-on, engaging activities (Van de Walle, 2004:308) and transformation geometry (or motion geometry, that is translation [slide], reflection [flip], rotation [turn] and dilation [stretch or shrink] (Geddes & Fortunato, 1993:212).

It appears that in order to master space and shape in the mathematics learning area, learners must be able to develop a multitude of reasoning skills.

Reasoning about spatial concepts

Space and shape involve connections with various other areas of mathematics. An understanding of measurement, proportional reasoning, algebra and integers, among others, is necessary to develop an understanding of space and shape (geometry). Van de Walle (2004:347) defines spatial sense as an intuition about shapes and the relationships among shapes. He argues that although 'a feel' for geometric aspects is implied in the definition, experiences with space and shape can develop spatial sense. This belief is consistent with research which states that understanding is built in geometry across the grades, from informal to more formal thinking (NCTM, 2000:41).

Cognitive development in the learning of geometry has been a major focus of research. Piaget argues that the development of learners' concept of space progresses through various stages of acquisition, representation and characterisation of spatial concepts (Piaget, Inhelder & Szeminska, 1960). He considers this development as a maturation process (Geddes & Fortunato, 1993: 200). The Van Hiele model, on the other hand, suggests different levels of thinking focusing on experience through different phases of learning (Van Hiele, 1984). These phases may be recursive and are not necessarily achieved in a linear Piagetian progression. Contemporary views (Van de Walle, 2004: 348) support the van Hiele levels of geometric thought which propose a five-level progression towards the understanding of spatial ideas. The model suggests a progression towards understanding spatial ideas (Geddes & Fortunato, 1993; Van de Walle, 2004:347-384; Troutman & Lichtenberg, 2003:410):

Level 0 represents the visual characteristics of a figure or judging shapes by their appearance. The argument that a square is a square because it looks like a square results in the ability to identify shapes that seem to be similar or alike.

Level 1 represents the ability to analyse shapes and to categorise them; in other words, shapes are classified. By focusing on a class of shapes, learners are able to think about what makes a square a square (four equal parallel sides, four right angles, congruent diagonals bisecting perpendicularly). Mastery at this level represents the understanding of the properties of shapes.

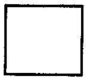
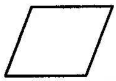
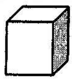
Level 2 addresses informal deduction, in the sense that observations go beyond properties themselves, and focuses on logical arguments about the properties. Thus, previously discovered properties are interrelated. The result of thought at this level is relationships among properties (comparison) of geometric objects.

Level 3 has to do with deduction indicating the ability of learners to examine more than just the properties of shapes. Relationships among shapes are deduced. Learners begin to appreciate the need for a system of logic that is based on a minimum set of assumptions and from which other truths may be derived. At this level, abstract thought is developed and intuition is substituted by logic.

Level 4 represents an advanced level of axiomatic systems and an appreciation of the distinctions and relationships between different axiomatic systems. Rigour in logic is evident. This level addresses geometry beyond the high school level of the National Curriculum Statement.

Since no simple universal theory for the teaching of geometry exists (Troutman & Lichtenberg, 2003:410), Van Hiele's research is used as basis for investigating geometry experiences in this study. A steady progress in learners' geometric reasoning is assumed in the exposition of the learning outcomes over different grades (RNCS, 2002; NCS, 2003, NCTM, 2000:41). Mastering space and shape implies the formation of concepts and relationships or internal representations that match external representations (Troutman & Lichtenberg, 2003:10). In other words, based on the exposition of the five levels above, concept formation starts with the development of vocabulary and the recognition of shapes towards the identification and association of characteristics. For successful matching of internal representations with external representations, meaning is involved as a building block: learners must know various characteristics of concepts in order to identify unique features. This conceptualisation of shape is illustrated in Table 1.

Table 1 Conceptual understanding of shapes

Names	Objects	Characteristics (properties such as)	Relationships (aspects such as)
Square		Right angles; equal and parallel sides; intersecting diagonals	A square is a special quadrilateral
Rhombus		Opposite angles are equal; diagonals bisect at right angles	A square is a special rhombus
Cube		Right-angled surfaces; three-dimensionality	A square is a subset of a cube

Spatial sense may be interpreted as follows (compare Table 1): learners must first attach some form of meaning to objects (identification of geometric figures) which are consequently ordered together in groups that make sense to the learner (analysing the attributes of geometric figures, such as mastering properties of angles, sides and diagonals). In order to do this, learners must be able to identify the characteristics and relationships of the objects (comparing geometric figures to see how they are alike and how they are different and being able to distinguish between sets and subsets of quadrilaterals). Based on this, learners progress towards conceptual understanding by applying various reasoning skills in being exposed to experiences with space and shape. This framework of concept formation forms a basis which may lead to the development of higher order reasoning skills involving deeper levels of cognition including a multitude of reasoning patterns or 'science terms' as envisioned by official education documents (RNCS, 2002; NCS, 2003; NCTM, 2000).

Spatial sense is investigated next by means of a case study.

Research design

Form of inquiry

This investigation focused on concept formation and spatial visualisation. An educational case study as a form of inquiry was investigated. Fouché and De Vos (1998:125) argue that a case study may involve any one research subject or a group of subjects. In this investigation a process of activities of a particular group of teachers and learners was studied. Both teachers and learners completed the same set of geometry activities. The resulting information was organised, analysed, and evaluated. The focus was on spatial and conceptual aspects in geometry. An account is provided of how teachers and learners responded to problems related to space and shape.

Respondents

Respondents represented two groups, namely, mathematics teachers and mathematics learners. The mathematics teachers were students enrolled in a specific qualification — an Advanced Certificate in Education: Mathematics Education (ACE Mathematics) with the objective of improving their content knowledge and skills in mathematics teaching. The specific module involved mathematics classroom inquiry as a means of action research with the aim of improving classroom practice. The number of teachers registered for the specific course was 33. Teachers responded to a measuring instrument comprising representative geometry questions on Grade 10 level. Not all students attended the workshops and 29 teachers completed the investigation. A profile of the teachers is provided in Table 2.

Table 2 Teacher profile (n = 29)

Gender	Male 41%	Female 59%
Teaching experience	Range $1 \le x \le 16$	Mean 6
Age	Range $26 \le x \le 56$	Mean 36
School type	Urban 79%	Rural 21%

These teachers, in turn, had to administer the same measuring instrument to ten Grade 10 learners in mathematics classrooms at their respective schools, totalling 29 schools in rural and urban parts of the central region of South Africa. These Grade 10 learners formed the second group. The number of learners was 290. Grade 10 learners' gender and ages are reflected in Table 3.

Table 3 Learner profile (n = 290)

Gender	Male 51%	Female 49%
Age	Range $14 \le x \le 23$	Mean 16
School type	Urban 79%	Rural 21%

The home language of respondents (teachers and learners) was as follows: 76% of respondents spoke Sesotho; 14% spoke Tswana and 10% spoke Xhosa (n = 319).

Sampling procedure

A purposive sample was used for selecting mathematics teachers (Fouché & De Vos, 1998:198). All teachers registered for the module of a specific year were used in the study. The selection of learners by the teachers was based on dimensional sampling or quota sampling (Bailey, 1994:95). Each teacher selected ten Grade 10 learners from their respective schools. The same relative number (five each) from male and female learners from each school was selected using stratified random sampling (Johnson & Christensen, 2004: 274).

Techniques for data collection and processing

The techniques employed to analyse the results were statistical computations involving Microsoft Office Excel and SPSS programs.

Pilot study

Before finalising the measuring instrument, geometry problems similar to the final measuring instrument were posed to groups of ACE Mathematics students enrolled in the same module of the three previous years. Further aspects of the pilot study are addressed under the heading validity and reliability.

Validity and reliability

The measuring instrument was developed over a period of three years. The tasks chosen for the research were selected for the reason that similar activities had been previously used to develop visualisation during studies by other researchers such as Fischbein (1987), Robichaux (2000), and Thornton (2000). The measuring instrument was implemented by the students as a tool for action research in a research module of the ACE Mathematics course. Content validity may be classified as judgmental in this study as the choice of activities of the measuring instrument depended on the judgment of the researcher. Modifications were made to enhance authenticity by means of changes in formulation. In order to measure concepts more accurately, questions were changed, reformulated and adapted to eliminate ambiguity and to ensure content validity. As a result, construct validity was also enhanced because the instrument was adapted to improve the measurement of theoretical constructs such as reasoning and skills. Certain spatial visualisation problems were substituted with more authentic problems in order to eliminate difficulties experienced by respondents in grasping meanings in lesser authentic contexts (a problem involving distances between a lighthouse and a passing ship were substituted with a more authentic problem). The questions on relationships of quadrilaterals were structured differently and the problems on spatial visualisation were altered as a result of the experiences in previous years. Another aspect that could have influenced the validity of the investigation was that the judgment of the teachers in selecting the dimensional sample of learners may have been too prominent. With regard to reliability, the preamble of changing the measuring instrument (pilot study) enhanced reliability in the sense that the quality of data measuring was improved.

Another factor that may have had an influence on the validity of contextual data was the fact that some of the teachers did not provide information regarding questions pertaining to past performance such as the symbol obtained and mathematics level of the school-leaving certificate in mathematics (only 16 out of a possible 29 teachers divulged information regarding school-leaving performance). However, information was checked by means of official registration records of the teachers registered for the course.

Ethical considerations

Ethical considerations in obtaining access to data were adhered to: permission from the Education Department was granted. Districts and schools where the research would take place were identified and approved by the Department of Education.

Procedures for the analysis of data

A series of activities were conducted and specific procedures were employed for administering the measuring instrument. Teachers' and learners' ability to work with space commenced with the classification of shapes and deductions about their interrelationships; the ability to apply proportional reasoning and to calculate areas of triangles and rectangles, while three-dimensional problems involved the ability to identify and analyse the different faces of a prism, as well as to identify geometric shapes of ordinary figures such as boxes or cubes. Visualisation of faces (sides) of shapes from different views — front/right or back/left — concluded the activities on spatial sense. The responses were evaluated and the results were graded. Two main categories were identified (shape and space). The selection of problems was based on the mathematics learning area. The category shape was subdivided into classification, symmetry, and relationships, while the subcategories for space were proportion, three-dimensional calculations, three-dimensional figures and spatial visualisation. The activities were classified into Van Hiele's levels and subsequently analysed.

Measuring instrument

The measuring instrument comprised a series of questions involving typical problems on space and shape based on a Grade 10 geometry level. Before finalising the measuring instrument, similar geometry problems were posed to groups of ACE Mathematics students enrolled in the same module of three previous years. Questions addressed problematic areas in geometry on Grade 10 level. Questions were based on learning outcome three (geometry) of mathematics as the learning area of the National Curriculum Statement (NCS, 2003). Questions, *inter alia*, included statements of positive identification of figures (true-false objective test items), as well as calculations based on shape and space. Abilities to distinguish between various dimensions of space and shape problems were evaluated. A summary of the measuring instrument is provided in Table 4 indicating the type of geometry involved; an example of a

Table 4 Summary of the measuring instrument

Category	Example	Van Hiele level	Level indicator
A classification of quadrilaterals	Learners have to state whether true or false: Every parallelogram is a rhombus.	Level 1	Classification (Analysis)
A simple understanding of symmetry	Learners have to draw all the symmetrical lines in the figure provided.	Level 0	Identification (Visualisation)
Relationships between quadrilaterals	The relationship between all the types of quadrilaterals could be explained by a diagram. In the given diagram an example is provided by an arrow indicating that a parallelogram is a special type of quadrilateral. Learners are required to complete the diagram by drawing arrows to indicate special relationships. (The different possibilities from which to choose are provided.)	Level 3	Relationship (Deduction)
Recognising proportions	Six equally spaced parallel lines are intersected at three points, A, B and C. The distance between two points B and C is given and learners are required to calculate the distance between B and C.	Level 2	Comparison (Informal deduction)
Three-dimensional calculations	Learners have to calculate the surface area of a three-dimensional triangular figure which has a right-angled base.	Level 2	Comparison (Informal deduction)
Recognising geometric shapes	Learners are asked to select from five possibilities what region is represented by a flat surface if it is cut and folded together.	Level 0	Identification (Visualisation)
Spatial visualisation	A picture of a building, represented by various cubes, drawn from the front-right corner is provided and learners have to find the back view.	Level 3	Relationships (Deduction)

related problem is presented which is followed by the corresponding Van Hiele level and level indicator of the problem.

The focus of the first three groups of problems was on distinctions between certain geometric figures (shape) while the remaining problems pertained to shape.

These components were addressed, evaluated and graded separately. Subsequently, results on space and shape components were recorded. The results are analysed and discussed here.

Analysis and discussion of results

The following data pertaining to achievement levels are provided in order to analyse the problem areas relating to certain geometric figures, as well as problems experienced in mastering spatial perspectives. Means are provided for shape and space, as well as overall means. (Totals for data are 10, in other words, a number of 5.8 implies 58%). Figure 1 reflects the results of the activities of teachers (n = 29) and learners (n = 290) based on shape.

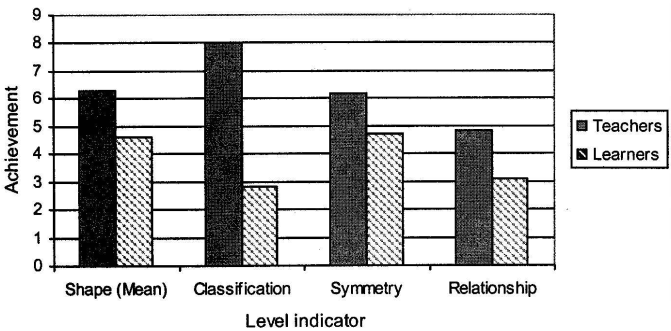


Figure 1 Conceptual understanding of Shape

Figure 2 indicates the results of the activities of teachers (n = 29) and learners (n = 290) based on space.

Teachers' and learners' ability to work with shapes included a classification of shapes and deductions about their interrelationships. In general, teachers' results were only slightly higher than learners' results.

The main problem areas concerned spatial aspects, specifically in dimensional reasoning (0.7 for teachers and 0.4 for learners). This was followed by proportional reasoning (2.0 for both teachers and learners) and spatial visualisation (2.2 for teachers and 1.6 for learners).

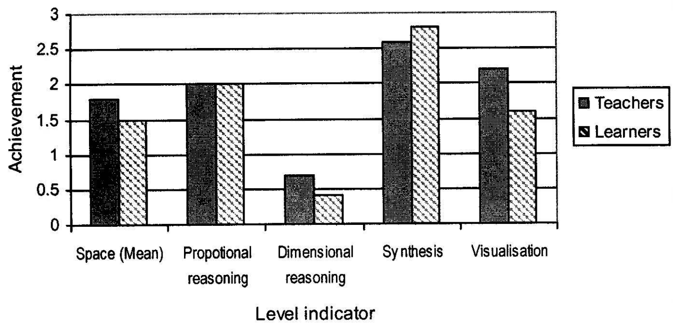


Figure 2 Conceptual understanding of Space

The results for proportional reasoning were the same for both groups and in one instance the mean for learners was greater than that of the teachers where the activity required a synthesis from two-dimensions (a flat surface) to three dimensions (a cube).

It was indicated earlier in Table 2 that the development of spatial sense progresses from conceptual identification to relevant deductions. In this investigation concept formation of shape and space (compare Table 2) was represented by:

- a simple understanding of symmetry and the recognition of geometric shapes (level 0);
- the identification and classification of quadrilaterals (level 1);
- the recognition of proportions and three-dimensional calculations (level 2);
- relationships among concepts (quadrilaterals in this context) and the ability to develop a logic system to address dimensional problems relating to shape and space (level 3).

If the problems are classified according to the Van Hiele levels (*cf.* classification in Table 2) the means in Figure 3 reflect mastery of teachers (n = 29) and learners (n = 290) on different levels.

The results indicated that a fair level of achievement was attained in identifying geometric figures (level 0 — 4.4 and 3.2 for teachers and learners, respectively) and analysing certain attributes of geometric figures (level 1 — 7.9 and 5.9 for teachers and learners, respectively). However, respondents did not perform very well on higher levels. Respondents' results revealed that problems were experienced in identifying results when geometric figures underwent change (three-dimensional examples on level 2 — 1.4 and 1.2 for teachers and learners, respectively). Similarly, in describing and justifying relationships among geometric figures, the results confirmed that problems

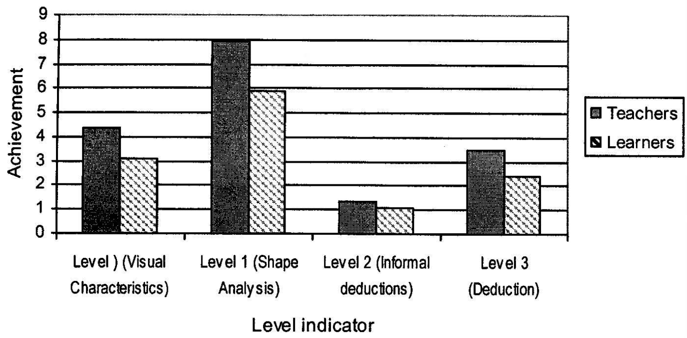


Figure 3 Achievement on Van Hiele levels

were also experienced on level 3 — 3.5 and 2.4 for teachers and learners, respectively.

Relationships between gender and achievement of teachers (n = 28) and learners (n = 280) are given in Tables 5a and 5b.

Table 5a Gender and achievement relationships of teachers

		Teachers	
		Shape	Space
Male teachers	Mean	6.0	2.7
(n = 12)	Standard deviation	2.5	1.8
Female teachers	Mean	6.1	1.4
(n = 16)	Standard deviation	1.0	1.4
Pearson correlation for space and shape, all teachers		$r = 0.28$	

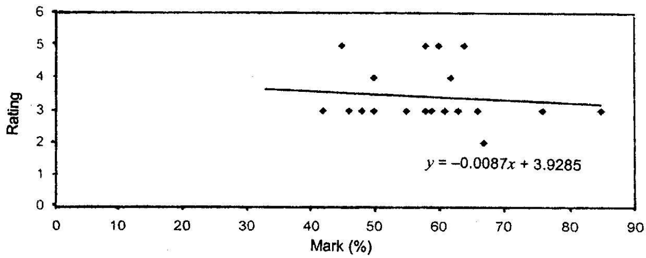
Table 5b Gender and achievement relationships of learners

		Learners	
		Shape	Space
Male learners	Mean	5.1	1.5
(n = 143)	Standard deviation	2.7	2.4
Female learners	Mean	4.2	1.5
(n = 137)	Standard deviation	2.4	2.2
Pearson correlation for space and shape, all learners		$r = 0.36$**	

** Correlation is significant at the 0.01 level (2-tailed)

A weak relationship between achievement in space and shape for teachers was observed ($r = 0.28$), whilst there was a significant correlation for learners in performance with regard to shape and space ($r = 0.36$). No clear distinctions between the performance by male and female respondents could be observed. However, in the case of learners, boys performed slightly better than girls. The small difference in achievement of teachers and learners was alarming.

In addition to the above results, teachers' personal views regarding their mathematics abilities were documented. The relationship between the way in which teachers rated their own mathematics abilities and their achievement is reflected in the scattergram in Figure 4. The results (in percentages) were considered as the independent variable while self-report on mathematics ability in teaching was the second interval variable.



Correlations

Teachers		Rate	Mark
Rate	Pearson Correlation	1	−.108
	Sig. (2-tailed)		.641
	N	27	21
Mark	Pearson Correlation	−.108	1
	Sig. (2-tailed)	.641	
	N	21	23

Figure 4 Relationship between teachers' achievement and self-rating

A negative relationship was evident between the two variables. As teachers' performance increased, a slight decrease in their own ability rating was observed. The regression line had a negative gradient. The Pearson correlation coefficient (−0.108) indicated a weak relationship between the variables. The implication was that teachers overestimated their own potential and abilities in comparison with their performance. This may have important practical outcomes for the teaching and learning of mathematics.

Conclusion

The investigation indicated that space and shape were problematic areas for both teachers and learners. Some of the problems experienced in space and shape in geometry classes were the following:

- Respondents had difficulty in representing characteristics of and relationships between two-dimensional and three-dimensional objects. Three-dimensional activities were specifically experienced as problematic.
- If geometric objects were placed in a variety of orientations and positions, respondents experienced problems in analysing and solving problems.
- Problems related to viewing objects from different angles revealed difficulties.
- Insight into volume and surface area needed attention: respondents did not fare well in being able to analyse a three-dimensional problem and being able to calculate its surface area correctly — the problem involved breaking a figure up into rectangles and right-angled surfaces.

The following findings emerged from the study:

- Correspondence between the achievement of teachers and learners could be observed: the achievement of both groups revealed a need to develop spatial sense in geometry.
- A strong correspondence between the performance of male and female respondents emerged from the study.
- An interesting phenomenon was that teachers overestimated their mathematics abilities. The role of the teacher should be taken into consideration concerning the content knowledge of mathematics as a discipline. Teacher subject knowledge is crucial. This study confirmed that such knowledge is a good predictor of learner achievement.
- It appeared that major emphasis is given in classrooms to the mastery of basic skills: calculation accuracy seemed to be preferred to application of procedures. Identifying geometric shapes in alternative or 'new' positions posed greater problems to teachers than to learners — a synthesis from two-dimensional (a flat surface to three-dimensions) was required. This tendency underlined the traditional emphasis on rote learning among teachers.
- Results did not indicate a gradual development from level 0 to level 3: respondents performed better on level 3 (deduction) than on level 2 (informal deduction) and on level 1 (shape analysis) than on level 0 (visual characteristics). This may give rise to a number of questions:
  - Is a 'feel' for space and shape a prerequisite for mastering geometry?
  - Will practice make perfect?
  - How may reasoning patterns be developed?
  - To what extent is it possible to correlate the activities of the measuring instrument with Van Hiele's theoretical model? 'Matching' may be possible; however, a perfect 'fit' remains theoretical and idealistic resulting in paper exercises and mere rhetoric.

Recommendations from the study are as follows:

- A 'feel' for geometric aspects may be developed by exposing aspiring mathematicians to a wide spectrum of experiences in geometry. Practical experiences with space and shape can develop spatial sense. Although an understanding of measurement and proportional reasoning is an advantage in developing mastery in geometry, experiences need not only be in formal Euclidean geometry. However, a firm Euclidean structure should be built up, eventually.

- Apart from a formal approach, other approaches to space and shape may be implemented to develop learners' thinking from informal to formal. Various approaches were identified in this investigation.

There is a need to develop the ability to apply proportional reasoning and to calculate areas of triangles and rectangles. Through investigating and implementing alternative approaches, reasoning abilities may be developed to produce conjectures about space and shape. This process of advancing from informal to formal thinking may consolidate a network of concepts and relationships formed by internal representations that mirror desired external representations.

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